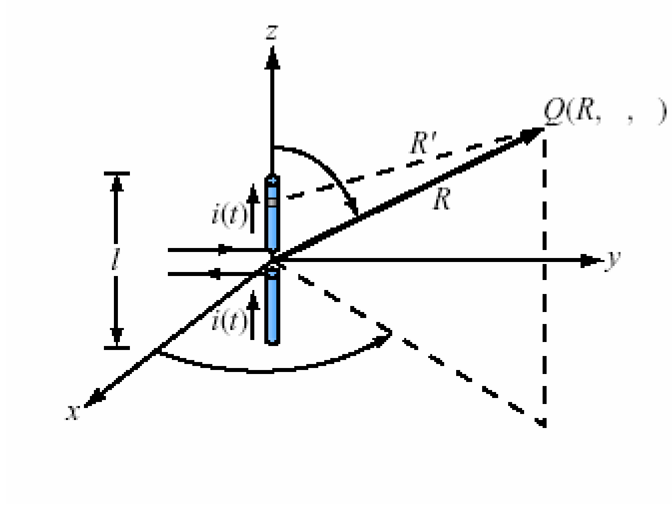
Ch7 Antenna and radiation

7.1 Dipole Antenna

A dipole antenna is one of the most fundamental antenna structures. When the antenna is very short compared to the wavelength, the radiation electromagnetic field form a short dipole antenna is given as:



$$H_\phi = \frac{I_0 \ell k^2}{4\pi} e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} \right] \sin\theta$$

$$E_R = \frac{2 I_0 \ell k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \cos\theta$$

$$E_\theta = \frac{I_0 \ell k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \sin\theta$$

(7.1.1)

When the observation point I svery far compared to the wavelength, i.e., $kR \gg 1$, (7.1.1) can be approximately given by:

$$E_\theta = \frac{j I_0 \ell k}{4\pi} \eta_0 \frac{e^{-jkR}}{R} \sin\theta$$

$$H_\phi = \frac{E_\theta}{\eta_0} \qquad (7.1.2)$$

7.2 Uniform array antenna

A radiation pattern of fundamental antennas such as a dipole antenna is fixed, and it cannot be modified easily. An array antenna is a system that comprises a number of radiating elements, generally similar, that are arranged and excited to obtain directional pattern. Fig.1 shows one of the fundamental configurations of an array antenna. The radiation element: antennas are oriented to z direction, so it has an omni directional radiation pattern in the x-y plane, and the elements are equally spaced and aligned on the y axis.

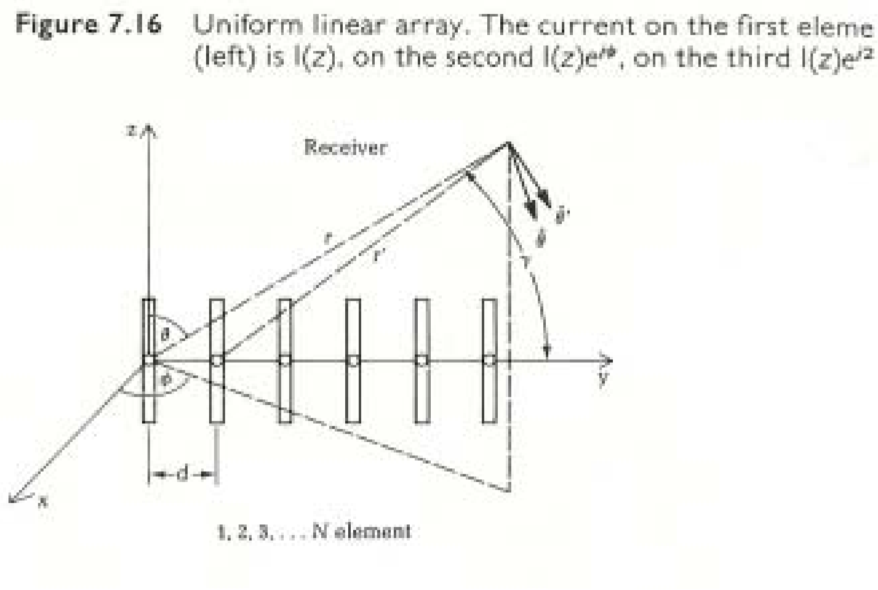


Fig.1 Uniform linear array

The current on the first element is $I(z)$ and the second element is $I(z)e^{j\psi}$, and the third is $i(z)e^{j2\psi}$ and so on.

Using the principle of superposition, we can obtain the total E field of the array antenna by adding the field of radiation field from each dipole antenna.. The radiation field from the element #1 is given by:

$$E_1 = \hat{\theta}\frac{jk\eta e^{-jkr}}{4\pi r}\sin\theta U(\theta) \qquad (7.2.1)$$

similarly, the radiation from the second element is:

$$E_2 = \hat{\theta}\frac{jk\eta e^{-jkr'}}{4\pi r}\sin\theta' U(\theta') \qquad (7.2.2)$$

Because the distance r and r' are approximately equal from the origin, we use the following approximation:

$$\begin{aligned}&\theta \approx \theta' \\ &r' \approx r - d(\hat{y}\bullet\hat{r}) = r - d\cos\gamma = r - d\sin\theta\sin\phi\end{aligned} \qquad (7.2.3)$$

(7.2.3) into (7.2.2) yields

$$\begin{aligned}E_2 &= \hat{\theta}\frac{jk\eta e^{-jkr}}{4\pi r}e^{jkd\sin\theta\sin\phi}\sin\theta e^{j\psi}U(\theta) \\ &= E_1\exp\left[j(kd\sin\theta\sin\phi+\psi)\right]\end{aligned} \qquad (7.2.4)$$

Generalizing this procedure, we obtain the radiation pattern from the N element array antenna as:

$$E_t = \hat{\theta}\frac{jk\eta e^{-jkr}}{4\pi r}\sin\theta U(\theta)\left\{1+\exp\left[j(\psi+kd\cos\gamma)\right]+\cdots+\exp\left[j(N-1)(\psi+kd\cos\gamma)\right]\right\} \qquad (7.2.5)$$

where

$$\cos\gamma = \sin\theta\sin\phi \qquad (7.2.6)$$

and where

$$U(\theta) = \int_{-h}^{h} I(z)\exp(jkz\cos\theta)dz \qquad (7.2.7)$$

is the radiation pattern of an dipole antenna.

The terms in the brackets of (7.2.5) are of the form

$$\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x} \qquad (7.2.8)$$

with the identification $x = \exp\left[j(\psi+kd\cos\gamma)\right]$ and the total E field of an array can be give as:

$$E_t = \left[E_e(\theta)\right]\left[F(\theta,\phi)\right] \qquad (7.2.9)$$

where

$$E_e(\theta) = \hat{\theta}\frac{jk\eta e^{-jkr}}{4\pi r}\sin\theta U(\theta) \qquad (7.2.10)$$

$$F(\theta,\phi) = 1+\exp\left[j(\psi+kd\cos\gamma)\right]+\cdots+\exp\left[j(N-1)(\psi+kd\cos\gamma)\right] = \frac{1-\exp\left[jN(\psi+kd\cos\gamma)\right]}{1-\exp\left[j(\psi+kd\cos\gamma)\right]} \qquad (7.2.11)$$

$E_e(\theta)$ is the field by a dipole, and $F(\theta,\phi)$ is called array factor. The magnitude of the electric field is given as:

$$|E_\theta| = |E_e||F| \qquad (7.2.11)$$

where

$$|F| = \left|\frac{1-\exp[jN(\psi + kd\cos\gamma)]}{1-\exp[j(\psi + kd\cos\gamma)]}\right| = \left|\frac{\sin N\left(\frac{kd\cos\gamma+\psi}{2}\right)}{\sin\left(\frac{kd\cos\gamma+\psi}{2}\right)}\right| \qquad (7.2.12)$$

or by putting $u = kd\cos\gamma + \psi$ we ca rewrite (7.2.12) as

$$|F(u)| = \left|\frac{\sin N\left(\frac{u}{2}\right)}{\sin\left(\frac{u}{2}\right)}\right| \qquad (7.2.13)$$

Here u can take a value

$$-kd \le u \le kd \qquad (7.2.14)$$

and it defines the visible range,

which means the range of angle, which really appears.

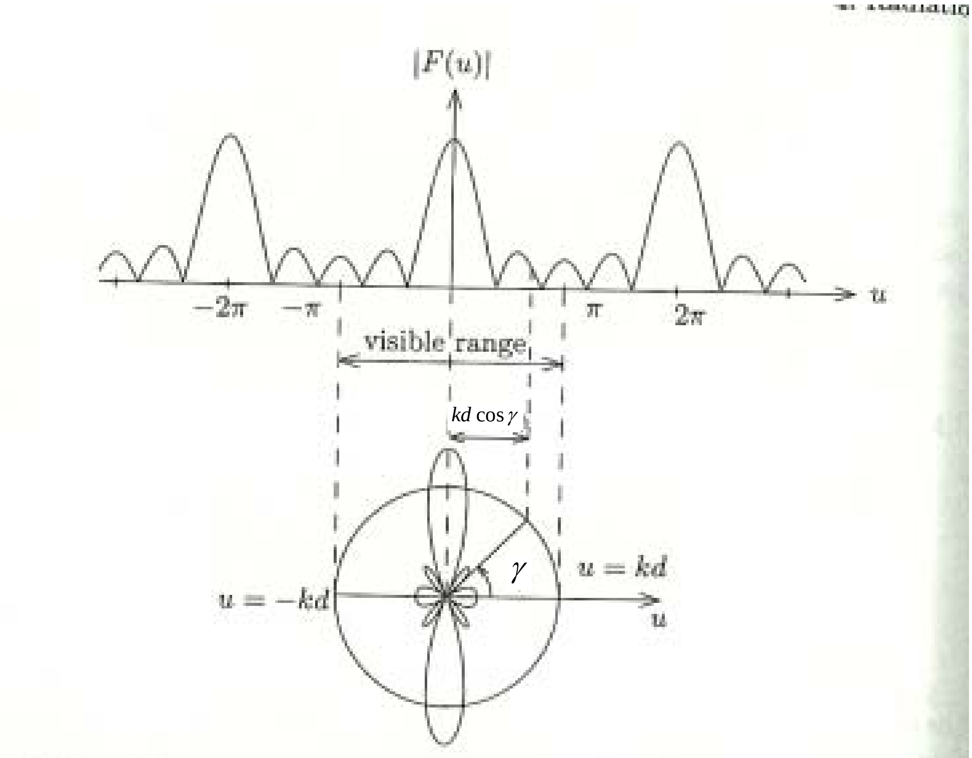


Figure 4.4.2 Array factor and radiation pattern for $N = 5$ and $kd = \pi$.

By adjusting the value of kd, which is the array spacing and ψ, which is the phase difference. we can change the array factor.

Case I: $d = \lambda/2, kd = \pi, \psi = 0, |E_\theta| \sim \left|\cos\left(\frac{\pi}{2}\sin\phi\right)\right|$

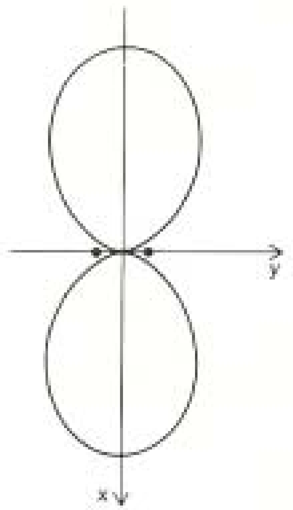
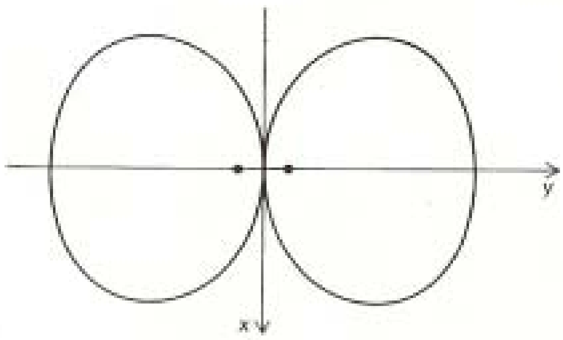


Figure 7.17 Radiation pattern of a two-element array with $d = \lambda/2$, $\psi = 0$.

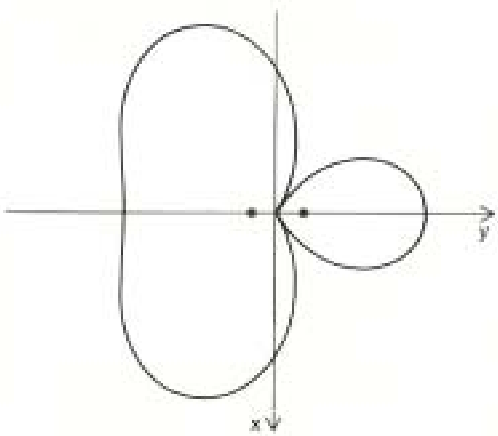
Case II: $d = \lambda/2, kd = \pi, \psi = \pi, |E_\theta| \sim \left|\cos\left(\frac{\pi}{2}\sin\phi + \frac{\pi}{2}\right)\right|$

Figure 7.18 Radiation pattern of a two-element array with $d = \lambda/2$, $\psi = \pi$.



Case III: $d = \lambda/2, kd = \pi, \psi = \frac{\pi}{2}, |E_\theta| \sim \left|\cos\left(\frac{\pi}{2}\sin\phi + \frac{\pi}{4}\right)\right|$

Figure 7.19 Radiation pattern of a two-element array with $d = \lambda/2$, $\psi = \pi/2$.



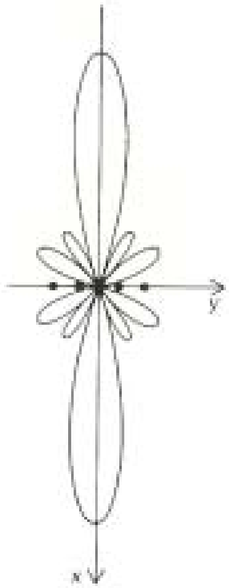


Figure 7.23 Radiation pattern of a four-element array with $d = 0.75\lambda$, $\psi = 0$.

If we want to have a sharp radiation pattern, we should have larger kd, however, at the same time, we can include the main lobe which is next to the center main lobe in the visible range. It is called a grazing lobe, which is normally not suitable for an array design.

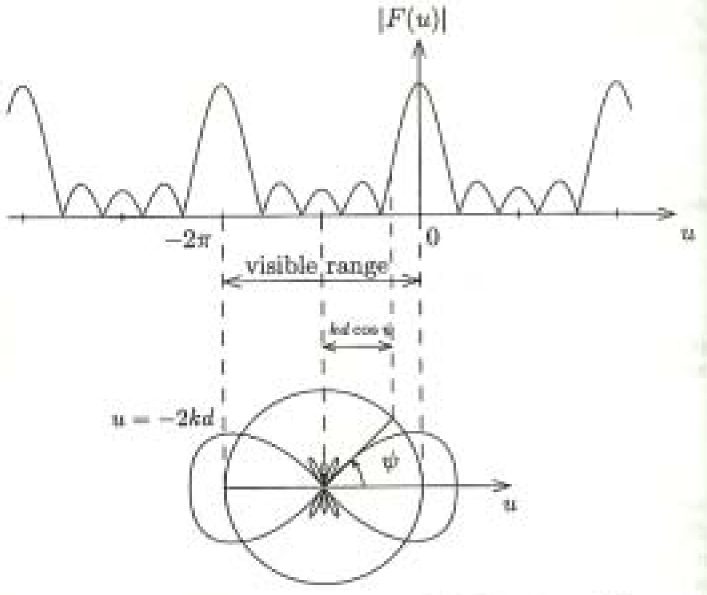


Figure E4.4.1.1 Visible range for $N = 5$, and $\alpha = kd = \pi$.

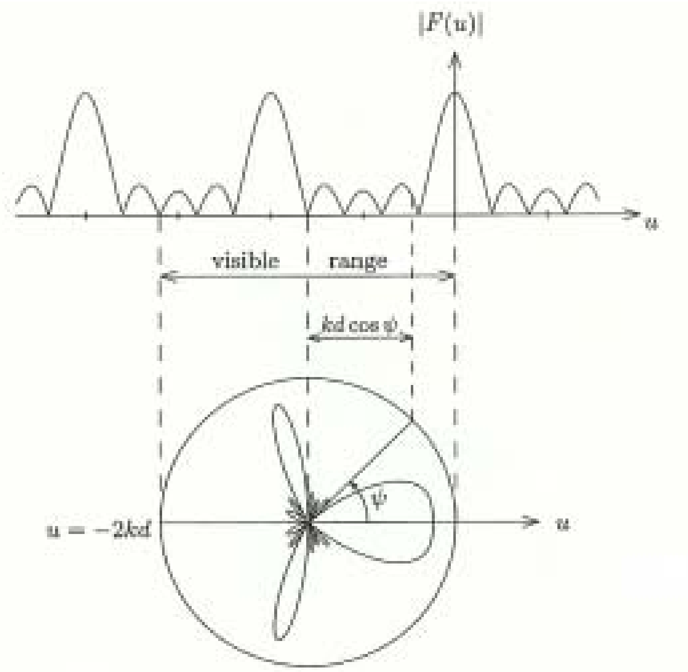


Figure E4.4.1.2 Visible range for $N = 5$, and $\alpha = kd = 8\pi/5$.

7.3 Radiation by large aperture antennas

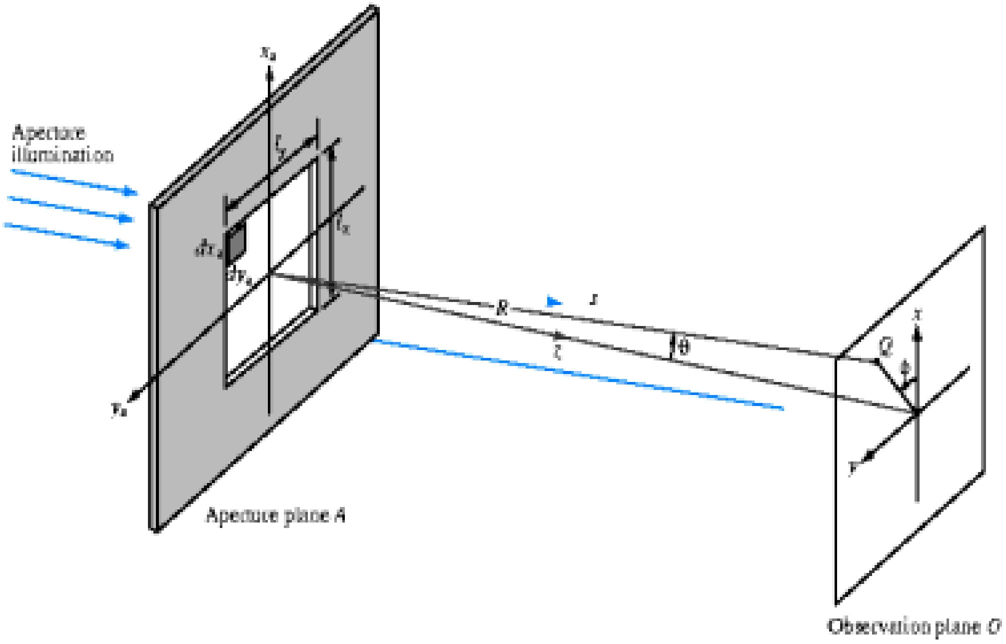


Figure 9-22

Fig.1.1 Antenna aperture (Ulaby 2001)

The antenna radiation pattern at far region, from an aperture shown in Fig.4.1 can be given by assuming the field strength on the aperture as:

$$E_a(x_a, y_a) = \begin{cases} E_0 & \text{for } -\ell_x/2 \le x_a \le \ell_x/2 \text{ and } -\ell_x/2 \le y_a \le \ell_x/2 \\ 0 & Otherwise \end{cases} \quad (7.3.1)$$

Then the radiation pattern in x-z plane ($\phi = 0$)is given by:

$$\begin{aligned} h(\theta) &= \int_{-\ell_y/2}^{\ell_y/2} \int_{-\ell_x/2}^{\ell_x/2} E_0 \exp(jkx_a \sin\theta) dx_a dy_a \\ &= E_0 \ell_x \ell_y \frac{\sin(\pi \ell_x \sin\theta/\lambda)}{\pi \ell_x \sin\theta/\lambda} \\ &= E_0 \ell_x \ell_y \sin c(\pi \ell_x \sin\theta/\lambda) \end{aligned} \quad (7.3.2)$$

Then the power density of the radiation pattern is given by

$$S(R,\theta) = S_0 \sin c^2(\pi \ell_x \sin\theta/\lambda) \quad (x - z \quad plane) \quad (7.3.3)$$

By the definition of radiation pattern, it is given by:

$$F(\theta) = \frac{S(R,\theta)}{S_{\max}} = \sin c^2(\pi \ell_x \sin\theta / \lambda) \qquad (7.3.4)$$

By using (1.4) we can find the beamwidth of the radiation pattern by solving

$$F(\theta_2) = \sin c^2(\pi \ell_x \sin\theta_2 / \lambda) = 0.5 \qquad (7.3.5)$$

$$2\theta_2 \approx 2\sin\theta_2 = 0.88\frac{\lambda}{\ell_x} \approx \frac{\lambda}{\ell_x} \qquad (7.3.6)$$

7.4 Radar resolution

When an antenna is equipped on a spacecraft or an airplane, the radiation pattern from this antenna determines the area of the radar signal. This area is called "foot print" of radar system, and it determines the resolution of radar system.

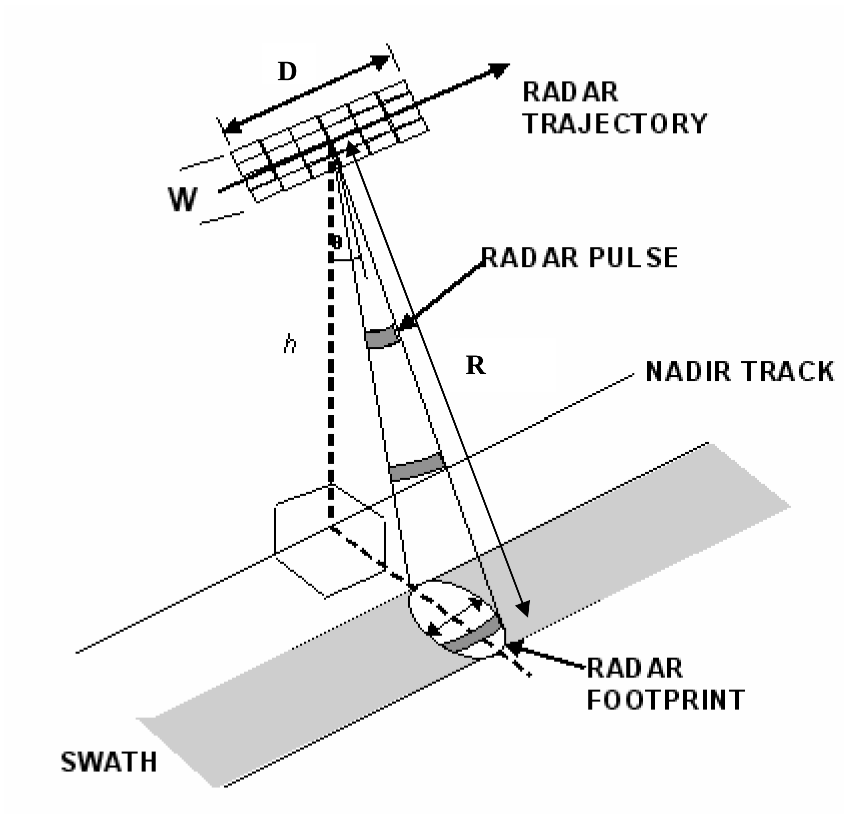
Radar range resolution ρ_{rg} depends on the pulse width τ and it is given by the frequency bandwidth W and is given as:

$$\rho_{rg} = \frac{c\tau}{2} = \frac{c}{2W} \qquad (7.4.1)$$

In the azimuth direction, the antenna angle of radiation pattern is approximately given by (7.3.6) and the actual length of the azimuth radiation pattern is

$$\rho_{az} = 2\theta_2 R \simeq \frac{\lambda}{D} R \qquad (7.4.2)$$

where D is the size of the antenna aperture and R is the range from the antenna to the radar foot print on the ground surface.



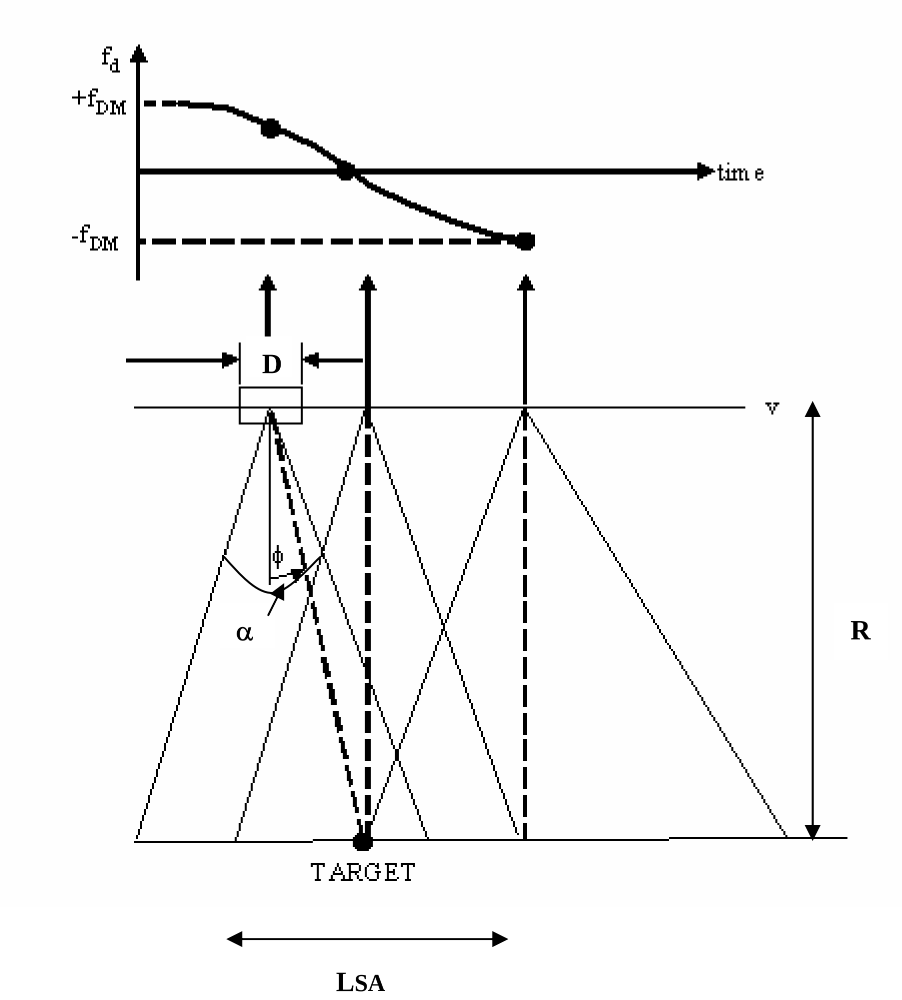
The definition above is applied directly for the antenna, and it is called a real aperture radar.

7.5 Synthetic Aperture Radar (SAR)

On the contrary to the real aperture radar, synthetic aperture radar (SAR) utilizes the radar information during the certain period of flight. When the radar target is stationary and the radar acquires the data while flying the flight path length of L_{SA}, the azimuth resolution of the total data sets are given as;

$$\alpha_{SAR} \simeq \frac{\lambda}{2L_{SA}} \qquad (7.5.1)$$

The factor 2 accounts for the effect of sequential emission of the elements of the synthesized antenna in the case of SAR. The phase difference between equally spaced elements of the synthetic aperture is over the two-way-path difference, and thus it is twice that of a conventional real aperture antenna, where the elements transmit simultaneously.



Using the result obtained for the real aperture antenna, we can derive:

$$L_{SA} = \alpha R = \frac{\lambda}{D} R \quad (7.5.2)$$

and substituting (7.5.2) into (7.5.1) gives:

$$\alpha_{SAR} = \frac{D}{2R} \qquad (7.5.3)$$

and the azimuth resolution is given by:

$$\rho_{AZ} = \alpha_{SAR} R = \frac{D}{2} \qquad (7.5.4)$$